

**TEAM IMPROVEMENT IN THE UEFA CHAMPIONS LEAGUE:  
AN APPLICATION OF DATA ENVELOPMENT ANALYSIS**

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## **ABSTRACT**

Football teams, when considered economic organizations, are expected to achieve their objectives without consuming excess resources. Put another way, they must produce their products and services efficiently.

In this study, we calculate efficiency on the playing field for teams in the UEFA Champions League between 2003 and 2012. We do this not only with an aim of separating the efficient and inefficient teams, but also to propose a way for inefficient teams to become more efficient. To this end, we use Data Envelopment Analysis to calculate efficiency ratios. From those results, we calculate improvement percentages that we then use as a basis for team-management recommendations.

We find that no single input is to blame for inefficiency. The general recommendation for inefficient teams is to use fewer attacking plays.

JEL Codes: M10, M21

Keywords: Efficiency, football, Data Envelopment Analysis, percentage of improvement, UEFA Champions League.

## 1. INTRODUCTION

Economists have long studied the sports sector. Rottenberg (1956), Neale (1964), Jones (1969), Sloane (1969) and El-Hodiri and Quirk (1971) are pioneers in this line of research, initiating studies in fields that continue to be under investigation. Rottenberg (1955), Sloane (1969), and El-Hodiri and Quirk (1971) study the unique characteristics and effects of recruiting athletes, for example; Jones (1969) analyzes which variables represent the objectives of sports clubs, and Neale (1964) focuses on the peculiarities of the sports sector. These issues are still the subject of recent research. For example, Szymanski (1999) studies the market for football players in England, Késenne (2005 and 2006) deals with the objective functions of sports clubs, and Brook (2005) questions what sports teams produce.

The scope of sports economics, however, currently embraces other topics, such as the operation of tournaments (Noll, 2002; Ross and Szymanski, 2002; Szymanski and Valletti, 2005), television broadcasting revenues (Jeanrenaud and Késenne, 2006), finance (Andreff, 2006), and competitive balance (Szymanski and Késenne, 2004).

Nevertheless, researchers rarely consider the performance of sports clubs or the ways to evaluate such performance. Specifically, any economic organization can be evaluated from an efficiency standpoint, and sports clubs are no exception. Because efficiency is related to a lack of waste in producing output, we can use research approaches for production to calculate it. For instance, Szymanski and Smith (1997) and Szymanski (2012) study the financial performance of English professional football. They establish a behavioral model for football club owners that specifies the production function as the league position depending on wages paid to players. Menéndez, Bello-Orgaz, and Camacho (2013) also present a behavioral model but for football players and team strategies. Their study of the 2010 FIFA World Cup applies extraction techniques to data regarding plays in the field. Zak, Huang, and Siegfried (1979) is among the first studies to investigate directly efficiency in sport teams.

Research in this area primarily uses Data Envelopment Analysis (DEA) and stochastic frontiers as calculation tools. The first group includes studies such as Mazur (1994) and Anderson and Sharp (1997) on baseball; Fizel and D'Itri (1997) on basketball; Collier, Johnson, and Ruggiero (2010) on American football; and Haas (2003), Guzmán (2006), and González-Gomez and Picazo-Tadeo (2009) on football. On the other hand, Carmichael and Thomas (1995) and Dawson, Dobson, and Gerrard (2000a and b) use stochastic frontiers to calculate efficiency ratios for football teams.

Because few studies examine the efficiency of football teams in the UEFA Champions League, we fill the gap. Efficiency-frontier models, both DEA and stochastic frontier analysis, calculate efficiency ratios for each organization in their study samples. These ratios are typically the percentage by which the organization should reduce its resource consumption in order to become efficient. The sample for this study includes European football teams that participated in the UEFA Champions League during the seasons between 2003 and 2012.

The main advantage of DEA in comparison with stochastic frontiers is that it does not require a specified production function. DEA estimates the isoquant as an envelope of the data of inputs used and outputs obtained by the sample, solving as many linear programming problems as organizations in the sample. Applying these ideas to football teams, the plays during the game are the factors of production, which in turn depend on exercises, preparation, and other factors carried out in the training sessions. We could say that players continuously adapt to moves their teammates and opponents make. It is difficult to include that in a formula estimated using econometric techniques, however, which is why we choose DEA to calculate efficiency in football teams in the UEFA Champions League.

For these reasons, the number of plays in each game is a variable that is difficult to control. Therefore, the standard recommendation of efficient-frontier models that inefficient organizations reduce the amount of resources they use does not apply to football teams.

Similar to Cooper, Ruiz, and Sirvent (2009), who use DEA to calculate the improvement percentage, we use the improvement percentage as an indicator of the differences in the overall style of play. Our goal is to use this variable as the basis to propose changes in inefficient teams; that is, rather than reducing the number of plays, our suggestions will be oriented to changing existing tactics.

Although Cooper, Ruiz, and Sirvent (2009) calculate efficiency applying DEA, we can also calculate percentage improvement using stochastic frontiers. Nevertheless, DEA has two additional advantages. First, it is easy to calculate the amounts of required inputs and obtained outputs for the reference unit. Second, in the case of stochastic frontiers, an improvement percentage higher than the unity could be possible because deviations from the frontier may be due to random effects.

This study is structured as follows. Section 2 establishes the representative production function of the football teams. Section 3 shows the DEA calculations

proposed to calculate the efficiency, as well as the meaning and interpretation of the improvement percentages. The results and conclusions then follow.

## **2. PRODUCTION FUNCTIONS FOR FOOTBALL TEAMS**

Calculating organizational efficiency using the frontier methods comes from the notion that efficient units are situated on the isoquant and the remainder are classified as inefficient. The distance from the isoquant indicates the degree of inefficiency. In addition, the isoquant is derived from production possibilities, which indicate the maximum amount of final product that can be obtained for each combination of productive resources (Farrell, 1957). This notion of maximization is what makes the isoquant a frontier: it is possible to find organizations that are situated above it (i.e., they use more resources than necessary to obtain a certain amount of product), but there can never be organizations below it.

Frontier methods for calculating efficiency estimate the isoquant (or frontier) from actual observations. Therefore, input and output variables should be those that would appear on the isoquant representing the production function of the sector analyzed. It should also be noted that the quantity of resources used is the organization's decision and that a normal resource is one that increases output if a greater amount of it is employed.

Moreover, the production possibilities and their corresponding isoquants are formed by different combinations of quantities of resources. Accordingly, the differences in the characteristics of the resources (such as the skill of the workforce, ease of access to materials, etc.) are not captured in their representation nor in their characterization. It is precisely these differences, however, that explain why some organizations are on the isoquant and are therefore classified as efficient.

The literature provides a vision of the nature of the sports sector from an economics viewpoint that must be taken into account when specifying the production functions of sports teams.<sup>1</sup> In relation to this, two general issues recur. The first is regarding the aims of sports clubs. Studies by Cairns, Jennet, and Sloane (1986); Goddard (2009); and Késenne (2009), among others, argue that two common alternatives are the maximization of profit and maximization of victories, subject to the constraint of zero profit. However,

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<sup>1</sup> The majority of the citations and the ideas put forth refer to team sports, which are the object of study for this research. Generally speaking, the affirmations that follow would also apply to individual sports, although the variable "output" would no longer refer to the game but rather the type of competition (a race, for example).

they also propose a third option, which consists of maximizing a utility function that depends on the profit achieved, among other variables.

The second recurring issue refers to production, which is of interest in this study. As in any organization, sports clubs must achieve their proposed aims (financial profit, victory, or utility) by offering the market a product or service. Neale (1964) posits that sports companies produce an indivisible asset from separate processes in two or more companies, which leads him to conclude that from an economic point of view, the league as a whole is a company and that each game is the equivalent of a production plant of that company. Meanwhile, Brook (2005) states that the output of sports teams is the game itself. Essentially, the goal of the production process is to offer a show in the form of a tournament or match, for example.

In this respect, studies that assess sports teams use victories as the representative variable of the output;<sup>2</sup> variables related to the talent and performance of the players and coach are input variables (Hadley, Poitras, Ruggiero, and Knowles, 2000; Dawson, Dobson, and Gerrard, 2000b; Borland, 2009).

However, more specifically, Schofield (1988), Carmichael and Thomas (1995), and Carmichael, Thomas, and Ward (2000) claim that the production function of sports teams consists of two phases. In the first one, the skills, physical fitness, and player experience, as well as the manager's skills, are the resources that through training become the plays executed on the field. In the second stage, these plays are the inputs that, during the game, create results (measured by games won, points scored, etc.). This study uses this approach to calculate and evaluate football teams in the UEFA Champions League between 2003 and 2012. The analysis focuses on the second stage of the production process (i.e., efficiency on the field).

Examples of studies that calculate the efficiency of multiple-stage production processes include Lovell, Walters, and Wood (1994), Keh and Chu (2003), Sexton and Lewis (2003), and Medina-Borja and Triantis (2011); although the samples in each of the studies are from different economic sectors, they all take into consideration that the outputs of one stage are the inputs for the following stage. Along these lines, some

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<sup>2</sup> If sporting success is the goal of a club, it directly follows that the representative output variable is the number of victories. For two reasons, this does not change if the club's aim is financial profit or utility: first, because the input and output variables represent only their transformation into products (i.e., they focus more on technical rather than management issues), and second, because sporting success is expected to lead to increased revenue (through stadium attendance and via broadcasting rights) and therefore increased profit.

studies propose more sophisticated methods for calculating efficiency values for the overall production process. For example, Lewis, Lock, and Sexton (2009) and Chen, Cook, and Zhu (2010) consider the optimal values of intermediate-stage outputs as inputs. Also, Kao and Huang (2008 and 2011) propose calculations to break down the efficiency of the overall process into stages.

In any case, all of them select the input variables in the same way: raw materials, capital, and labor are inputs only in the first stage. In these studies, intermediate outputs obtained from the production-process stages are always taken as production factors for successive stages. Consequently, to calculate the efficiency of the second stage of the production process for football teams, only plays executed during games are incorporated as inputs, excluding the number of players or facilities used.

Because this study analyzes the UEFA Champions League, the output variable must represent competitive success. The format of the competition is as follows: 64 teams finishing in the top positions of the national league championships form the tournament. They are placed into 16 groups of four, forming the “group stage” in which each team plays another at home and away in a league format. The teams finishing first and second in each group enter the “knockout phase” in which each team is paired with another. The pairs play each other twice, once as the home team and once as the away team, and the winners progress. The process repeats through to the final, when one game decides the winner of the competition for that year.

Although teams participating in this competition are awarded points based on the results of each game in the group stage, in reality it is a knockout competition where the teams only play against the other teams within their group.<sup>3</sup> Frequently, teams do not move to the next stage even though they have more points than another team that does progress but belongs to another group. Therefore, if success in the UEFA Champions League is to win the final, and points don’t ensure that a team will move on to the following stage of the competition or reach the final, points can’t be the output variable. The same is true for scored goals. Consequently, if sporting success in this particular competition equals the ability to reach the next round of competition, the number of games played represents results.

On the field, teams use offensive plays to score goals and win matches and competitions, as well as use defensive actions to counteract the opposing team's attacks,

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<sup>3</sup> The groups are determined by lottery.

thus preventing their victory. The first phase (i.e., the training), is usually done without an audience, but the game itself is played in the presence of spectators, making the players' activities visible to the audience. However, this does not mean all of their activities are productive resources.<sup>4</sup>

A review of literature on the efficiency of football teams shows that researchers use frontier methodology (both DEA and stochastic frontiers) often, but most do not use plays as input variables. This is the case in Haas (2003); Haas, Kocher, and Sutter (2004); Guzmán (2006); Pestana-Barros, and Leach (2006, 2007); González-Gómez and Picazo-Tadeo (2009); or Kern, Shwarzmann, and Wiedenegger (2012).

However, Boscà, Liern, Martínez, and Sala (2009) and Sala-Garrido, Liern Carrión, Martínez Esteve, and Boscà (2009) use offensive and defensive plays as input variables for calculating efficiency via DEA. These two studies consider offense and defense two different production processes in football teams and thus study their efficiency separately. Our study agrees with this idea and uses only attacking plays as inputs. Football teams must of course make defensive plays to counter attacks by their opponents, but teams do not win by preventing goals; they win by scoring. In consequence, we could say that from the moves made by a team in the field, only attacking moves are directly related with obtaining output measured as sport result.

There is, however, a series of defensive plays that affect a team's success, and those are the plays made by the opposing team. However, they do not meet the second condition required of productive resources: control. Therefore we cannot consider them inputs for the second phase of the production process. Following Espitia-Escuer and García-Cebrián (2008), if the production function from which the isoquant, or frontier, is derived establishes a positive correlation between productive resources and the final product, then in the case of the football team, only offensive plays should be included.<sup>5</sup> Offensive plays directly increase the output obtained. Defensive plays could be

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<sup>4</sup> Manufacturing companies also perform activities outside the input transformation into the final product (e.g., marketing, distribution, and finance). These activities consume resources but are not taken into account when analyzing production-process efficiency.

<sup>5</sup> People often argue that the football teams with the best players are more efficient. For that reason a variable related to player qualities should be among the representative inputs of the production function. However, the isoquant, from which the efficient-frontier concept is derived, only collects quantitative variables. Therefore, variables that reflect qualitative differences have no place in the set of inputs. On the other hand, these differences may explain the different levels of efficiency. But for the sake of methodological rigor, they must be taken as explanatory variables of the efficiency ratios calculated at an earlier stage. This study not only identifies which teams are efficient; it also suggests modifications in play, which could include reconsidering players on which teams heavily rely.



interpreted, therefore, as an adaption to environmental circumstances that football teams must endure to achieve their objectives.

In summary, this study's efficiency results are based on attacking plays because a DEA formulation should only include resources directly related to the output (Triantis, 2004; and Cook, Tone, and Zhu, 2014). Efficiency of defensive actions in football teams could also be of interest, and separate calculations of efficiency for attacking and defensive processes would follow recommendations in Tofallis (1997)<sup>6</sup>. In the present article, it could be said that efficiency results are referred to the production of sport result from attacking plays.

The offensive plays include assists, centers into the area, corner kicks, entering the opponent's area, penalty shots awarded, and shots on goal. Descriptive statistics for these variables are shown in table 1.

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<sup>6</sup> Calculating efficiency for defensive plays requires more research about the representative variables for inputs and outputs in that process. A rigorous definition of the output, and in consequence for the inputs, concerning defensive plays is also needed.

**Table 1.** Descriptive statistics

		Assists	Centers	Corners	Opponent's area	Penalties	Shots	Games
<b>2003-4</b>	<b>Maximum</b>	115	360	73	399	2	181	13
	<b>Minimum</b>	34	105	17	115	0	53	6
	<b>Average</b>	63.125	190.313	38.781	205.156	0.656	100.281	7.813
	<b>Standard D</b>	22.954	69.721	15.622	76.714	0.701	38.541	2.278
<b>2004-5</b>	<b>Maximum</b>	105	330	76	358	4	162	13
	<b>Minimum</b>	23	91	14	96	0	46	6
	<b>Average</b>	55.969	182.313	36.500	196.719	0.938	99.719	7.813
	<b>Standard D</b>	23.549	70.761	15.381	75.923	1.243	38.468	2.278
<b>2005-6</b>	<b>Maximum</b>	110	327	67	413	3	217	13
	<b>Minimum</b>	29	102	19	122	0	51	6
	<b>Average</b>	57.188	189.719	37.094	212.813	0.781	103.281	7.813
	<b>Standard D</b>	22.103	62.207	13.388	77.529	1.099	42.083	2.278
<b>2006-7</b>	<b>Maximum</b>	135	410	73	484	5	209	13
	<b>Minimum</b>	25	121	19	127	0	50	6
	<b>Average</b>	67.281	201.875	36.750	229.125	0.531	105.563	7.813
	<b>Standard D</b>	26.934	75.482	12.753	88.850	1.107	42.395	2.278
<b>2007-8</b>	<b>Maximum</b>	141	418	90	484	3	216	13
	<b>Minimum</b>	24	108	16	117	0	44	6
	<b>Average</b>	67.719	209.813	39.719	240.875	0.750	107.344	7.813
	<b>Standard D</b>	30.657	79.869	19.155	100.010	0.803	46.316	2.278
<b>2008-9</b>	<b>Maximum</b>	143	444	89	509	4	227	13
	<b>Minimum</b>	25	103	16	116	0	49	6
	<b>Average</b>	65.281	216.781	40.344	243.313	0.563	104.469	7.813
	<b>Standard D</b>	28.130	87.977	18.811	106.420	1.014	45.912	2.278
<b>2009-10</b>	<b>Maximum</b>	145	384	98	468	2	204	13
	<b>Minimum</b>	21	91	16	100	0	34	6
	<b>Average</b>	68.656	189.469	38.531	214.563	0.563	102.844	7.813
	<b>Standard D</b>	30.108	73.894	16.963	88.183	0.716	45.345	2.278
<b>2010-11</b>	<b>Maximum</b>	162	282	73	342	4	221	13
	<b>Minimum</b>	33	86	12	101	0	46	6
	<b>Average</b>	72.531	168.969	39.625	198.625	0.906	105.781	7.813
	<b>Standard D</b>	33.893	55.709	17.705	71.392	1.118	47.123	2.278
<b>2011-12</b>	<b>Maximum</b>	192	286	85	383	6	255	13
	<b>Minimum</b>	32	72	15	85	0	54	6
	<b>Average</b>	75.719	156.375	38.906	190.688	1.094	107.500	7.813
	<b>Standard D</b>	39.064	56.138	16.717	75.291	1.279	53.007	2.278

Source: OPTA Sports.

Given the rivalry between teams in football matches, every action during a game could be interpreted as a reaction. So, it is necessary to establish a criterion for assigning plays. A common feature among offensive plays is that they start when a team keeps the ball and can be converted into a goal. Although assists, centers, entering the opponent's area, and shots on goal are actions taken exclusively by the players of the team under study, corners and penalties are a result of actions taken by the opposing team (corner kicks occur when the opposing team is the last to touch the ball before it crosses the goal line, and penalties occur when the opposing team fouls inside its own penalty area). Because the other team's actions create these events, the team under study has less control over the resource quantities it uses. However, we keep these actions as part of the representative variables for inputs because they could result in goals. In our recommendations, we account for this peculiarity regarding corner kicks in favor and penalties.

Finally, the purpose of this study is to propose ways for inefficient teams to improve. From a management perspective, the interest in knowing production-process efficiency levels resides in the fact that they indicate good use of resources. If a unit is efficient, it is producing something without wasting resources; an inefficient unit gets the same output by using more resources or obtains less output with the same resources.

This study calculates the efficiency of the second stage of the production process for football teams using plays made by players during games. Consequently, our recommendations refer to the style of play. However, these are not suggestions of a physical or technical nature to be taken into consideration exclusively by the coach; they are for any organization that wants to become more efficient. The ultimate economic aim of this study is to identify the resources wasted by inefficient European football teams so that, by improving their use, they can obtain each their objectives.

### 3. CALCULATING EFFICIENCY WITH DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) proposes to solve the following linear programming problem to calculate the efficiency of a unit  $i$  in a given sample:

$$\begin{aligned}
 &\text{Min.} && \lambda_i && (1) \\
 &\text{s.a.} && \lambda_i * x \geq u * X \\
 &&& y \leq u * Y \\
 &&& u \in R^+
 \end{aligned}$$

where:  $x$  is the quantity vector for inputs used by unit  $i$

$y$  is the quantity vector for outputs obtained by unit  $i$

$X$  is the quantity matrix for inputs used by the sample units

$Y$  is the quantity matrix for outputs obtained by the sample units

$u$  is the coefficient vector result of the problem

$\lambda_i$  is the efficiency ratio also obtained as a result of the problem

The formula assumes constant returns to scale and has an input orientation whereby  $\lambda_i$  is the rate at which the unit under analysis should reduce its resource use in order to be situated on the isoquant or frontier and be classified as efficient.<sup>7</sup> Therefore, all those units in the sample for which  $\lambda_i$  equals unity will be classified as efficient, and those with  $\lambda_i$  ratios below 1 are inefficient. Knowing the organizations' efficiency ratios prompts actions that allow the inefficient ones to situate themselves on the isoquant and thereby produce their current quantity of product with fewer resources, thus reducing their costs.

Solving problem 1 also provides information about the rate at which they should reduce their resource use. The values  $u*X$  and  $u*Y$  are, respectively, the amount of input that used and the amount of output obtained by an efficient unit (and which would therefore be on the frontier). This serves as a reference for the unit  $i$  analyzed.<sup>8</sup> This reference unit is what an inefficient organization should become in order to be efficient, and it is formulated in general as a linear combination of efficient units. In this case, it

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<sup>7</sup> For this study, the production function has only one output: the number of games played during the UEFA Champions League competition. The input orientation was deemed more appropriate because it makes more sense to recommend reducing the consumption of inputs than obtaining increases in output. The latter would imply that inefficient teams should increase the number of games they play, but the values that this variable can take are determined by the system of the competition.

<sup>8</sup> For a unit to be efficient, in addition to obtaining a  $\lambda_i$  ratio equal to 1 when solving the P1, exact input-amount  $u*X$  must be used to obtain output quantities  $u*Y$  (i.e., slacks  $(\lambda_i*x - u*X; y - u*Y)$  must be 0.

would be a point on the isoquant that does not correspond to any actual sample unit, but it could also be one of the units identified as efficient.

This information is used to calculate the improvement percentage proposed by Cooper, Ruiz, and Sirvent (2009), who evaluate the effectiveness of basketball players and are part of the DEA literature concerned with restricting the values for  $u$  that result from solving problem 1. Nevertheless, they also use DEA to shed light on two things: how efficient players achieve their ratios, and the aspects of the game that inefficient players can improve. For the first one, they use the information provided by  $u$ ; for the second, they use improvement percentage.

In this study, unlike in Cooper, Ruiz, and Sirvent (2009), we use an approximation of the production function that considers both resources and products; it also has an input orientation. Therefore, the formula to calculate the improvement percentage requires a modification to the one in Cooper, Ruiz, and Sirvent (2009).

First, our percentage of improvement calculation is based on the consumption of each of the productive resources. Consequently, the initial expression is the difference between the input used by the unit studied and that which would be consumed by its unit of reference:

$$\partial_{ri} = x_{ri} - \sum_{j=1}^k u^* x_{rj} \quad (2)$$

where:  $x_{ri}$  is the amount of input  $r$  used by unit  $i$ , and  
 $k$  is the number of companies in the sample

To facilitate interpretation of the results and to enable comparisons, we transform expression 2 as follows:

$$\text{Percentage of improvement} = \frac{\partial_{ri}}{x_{ri}} \quad (3)$$

This indicates the proportion by which the number of resource units  $r$  used by company  $i$  should be reduced for a company to be considered efficient. The improvement percentages for the inputs of inefficient units spotlight waste and what the possible explanation or consequences may be.

For inefficient football teams, improvement percentages behave in two ways. First, for teams that have similar values for all of their inputs, they imply that those teams must reduce the use of all of their productive resources. Their playing style is balanced relative to the reference team because their excess utilization of inputs is similar for all

the resources. The recommendation to these teams is to improve training activities in order to obtain better results with existing plays.

The second involves teams that have one or two productive factors in which the improvement percentage is significantly greater than for the other inputs. An increase in the efficiency of these teams might require a change in plays so that resources with higher improvement percentages would be reduced by a greater proportion. The training activities for these teams will thus revolve around designing new tactics. When an inefficient team frequently has a higher proportion of waste for a given resource, it is worth analyzing which could be the reasons of this situation and whether it affects the team's results.

#### 4. RESULTS

We calculate the efficiency ratio of European football teams in the UEFA Champions League between 2003 and 2012 using DEA. Although we could take as a whole the nine seasons for which data are available, we consider each of them a separate sample. In order to know if a serial temporal trend exists, we perform a Kruskal-Wallis test. The results reject the null hypothesis of equality in the ranking distributions, so productive circumstances change over time. Thus, efficiency calculations that use the whole sample are not appropriate.

Following Despotis (2002), the linear programming problems to be solved in this study and corresponding to each season in the sample have sufficient degrees of freedom because they comply with the rule established by the author, according to which:

$$k > \max \{m \cdot n, 3 \cdot (m+n)\} \quad (4)$$

where  $k$  is the number of observations,  $m$  is the number of outputs considered, and  $n$  is the number of inputs. A total of 32 teams participate each season in the UEFA Champions League and, in this study, the production function is represented by a final product and six productive resources.

The results are in tables 2 to 10. The efficiency ratios for each team are presented in the second column of each table. The percentage improvement that inefficient teams should apply to each input considered in this study are shown in columns 3-8. Finally, we calculate the range of variation between the maximum and minimum improvement

percentages for each team.<sup>9</sup> This is intended to analyze whether inefficient teams adopt a game similar to that of the reference values, and therefore whether the team should lower its consumption of all resources by a similar percentage or lower its consumption of a particular resource by a significantly higher percentage, which would indicate a change in game style.

A range of variation above 0.15<sup>10</sup> would represent of a situation in which increasing the efficiency of a football team would require a change in its style of play; the results for teams in this situation are shaded in the tables. Also, the teams that were winners (W) and finalists (F) in the competition each season are noted. Finally, teams that have not used the input of penalties awarded, and where the reference team also presents a null value, the improvement percentage is not determined. This suggests the team studied does not use this resource, which is precisely the recommendation made for it to be efficient.

As for any sample, the recommendations to football teams in the light of the efficiency results are directed at situating them on the efficient frontier. In this way, they may obtain results with fewer resources, as well as reduce their costs. Capital-intensive production processes with a physical or chemical transformation of material resources into products are controllable, so we expect that measures adopted to increase efficiency and savings produce the desired effects. However, production processes for football teams, particularly their second-stage processes, are labor-intensive and influenced by the circumstances of the game, which are different in each match. Therefore, even though teams may reduce their input use as a result of the efficiency calculations, the anticipated effects may not materialize and a value equal to unity is not reached.

This circumstance shouldn't invalidate the method or its results, because it is also true that DEA assigns the efficiency values from the data of a sample whose components should be acting in a similar environment. In this study, all the organizations in the sample are football teams in the UEFA Champions League and consequently, all of them use a production process subject to variability due to the

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<sup>9</sup> The improvement percentage for penalties is not included when calculating the variation range, because its value is not determined in many cases. Thus, the improvement percentage results for this input are mentioned separately.

<sup>10</sup> For each season and for each team, we calculate the standard deviation of the improvement percentages. We find that for all periods except 2008/2009, the value of these standard deviations is in the neighborhood of 0.15. Because the standard deviation measures dispersion around the mean, a range of variation of similar magnitude indicates substantial differences in the improvement percentages of the resources used by a given team.

changes in the conditions of each match.<sup>11</sup> This is why the recommendations in this paper are generalizations rather than exact prescriptions for reductions in specific resources quantities; that is, suggestions are based on the type of play put in practice by football teams.

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<sup>11</sup>In a league competition where all teams play each other at home and away, the influence of opposing teams on the production-process outcome would be more equal for the whole sample than in a UEFA Champions League competition. This is because the first phase is played with a league system, but only among the teams in a particular group, and the following phases are under a knock-out system. In fact, Espitia-Escuer and Garcia-Cebrián (2012) demonstrate that for the Spanish Professional Football League and the UEFA Champions League, the competition system influences teams' efficiency levels and results. However, one circumstance increases the homogeneity of teams in UEFA Champions League compared with those in the National League. In the UEFA Champions League, teams performed well in their respective national competitions the previous season, and they continue playing in their national championships at the same time as in the European competition. Meanwhile, leagues are made up of teams with more disparate circumstances, and just a few play simultaneously in international competitions.



**Table 2.** Efficiency ratios, improvement percentages, and variation ranges: Season 2003-4.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
AEK Athens	<b>100.00</b>							
Ajax	73.64	0.273	0.266	0.335	0.284	0.266	0.650	0.069
Arsenal	74.74	0.260	0.259	0.374	0.260	0.303	0.260	0.114
Bayern Munich	67.66	0.325	0.356	0.437	0.325	0.325	0.330	0.112
Besiktas JK	<b>100.00</b>							
Bruges	87.54	0.125	0.271	0.124	0.262	0.125	*	0.146
Celtic	87.75	0.114	0.123	0.113	0.129	0.114	0.180	0.016
Chelsea	66.26	0.260	0.227	0.235	0.249	0.259	*	0.033
Dep. La Coruña	80.15	0.199	0.213	0.211	0.198	0.199	1.000	0.015
Dynamo Kiev	68.86	0.323	0.317	0.488	0.311	0.418	*	0.176
FC Porto (W)	91.68	0.087	0.088	0.087	0.088	0.228	0.085	0.141
FK Partizan	95.61	0.034	0.036	0.035	0.046	0.034	*	0.012
Galatasaray SK	99.91	0.001	0.224	0.002	0.235	0.000	*	0.235
Inter Milan	74.86	0.252	0.280	0.359	0.251	0.310	*	0.108
Juventus FC	97.78	0.041	0.025	0.179	0.100	0.322	*	0.298
Lokomotiv Moscow	<b>100.00</b>							
Lyon	72.11	0.282	0.282	0.358	0.282	0.290	0.280	0.076
Manchester Utd	88.12	0.120	0.132	0.139	0.122	0.122	1.000	0.019
AC Milan	73.58	0.267	0.358	0.267	0.339	0.267	*	0.091
Monaco (F)	75.30	0.244	0.251	0.244	0.267	0.243	0.240	0.024
Olympiakos	83.24	0.167	0.168	0.351	0.168	0.381	*	0.214
O. Marseille	82.32	0.182	0.226	0.183	0.187	0.182	*	0.044
Panathinaikos	<b>100.00</b>							
PSV Eindhoven	<b>100.00</b>							
Celta Vigo	78.58	0.213	0.229	0.212	0.212	0.225	0.220	0.017
Rangers FC	<b>100.00</b>							
Real Madrid	78.69	0.216	0.216	0.368	0.260	0.217	0.250	0.152
Real Sociedad	83.95	0.163	0.227	0.415	0.231	0.187	1.000	0.253
RSC Anderlecht	92.99	0.069	0.071	0.158	0.117	0.070	1.000	0.088
Sparta Prague	<b>100.00</b>							
SS Lazio	66.93	0.332	0.360	0.356	0.330	0.331	1.000	0.030
VfB Stuttgart	80.14	0.248	0.225	0.284	0.203	0.219	0.200	0.081

\*: Indeterminate value for the percentage of improvement.

**Table 3.** Efficiency ratios, improvement percentages, and variation ranges: Season 2004-5.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
AC Roma	<b>100.00</b>							
Ajax	58.04	0.509	0.420	0.488	0.442	0.431	0.420	0.089
Arsenal	84.47	0.181	0.158	0.157	0.207	0.222	0.770	0.065
Bayer Leverkusen	63.82	0.574	0.363	0.363	0.366	0.495	*	0.211
Bayern Munich	52.15	0.501	0.506	0.502	0.482	0.477	1.000	0.029
Celtic	73.02	0.395	0.319	0.320	0.283	0.270	*	0.125
Chelsea	76.99	0.258	0.230	0.333	0.256	0.298	*	0.103
CSKA Moscow	63.59	0.446	0.364	0.602	0.385	0.365	0.720	0.238
Dep. La Coruña	82.14	0.179	0.374	0.433	0.344	0.179	*	0.255
FC Barcelona	49.26	0.651	0.505	0.530	0.512	0.541	0.505	0.146
FC Dynamo Kiev	68.66	0.521	0.313	0.320	0.322	0.489	*	0.207
FC Porto	71.35	0.622	0.293	0.449	0.288	0.589	*	0.334
FC Shakhtar Donetsk	87.62	0.361	0.124	0.414	0.161	0.324	*	0.290
Fenerbahçe	73.02	0.361	0.403	0.452	0.373	0.270	*	0.182
Inter Milan	85.66	0.363	0.142	0.162	0.186	0.294	0.140	0.221
Juventus FC	55.56	0.559	0.457	0.474	0.443	0.481	*	0.115
Liverpool (W)	74.38	0.255	0.290	0.385	0.289	0.255	1.000	0.130
Lyon	62.75	0.573	0.371	0.371	0.410	0.462	0.540	0.202
Maccabi Tel Aviv	<b>100.00</b>							
Manchester Utd	69.98	0.305	0.339	0.302	0.362	0.303	0.360	0.061
AC Milan (F)	61.52	0.525	0.395	0.515	0.390	0.384	*	0.141
Monaco	83.82	0.300	0.157	0.158	0.182	0.250	*	0.143
Olympiakos	<b>100.00</b>							
Panathinaikos	<b>100.00</b>							
Paris Saint-Germain	87.85	0.275	0.121	0.122	0.129	0.266	*	0.154
PSV Eindhoven	63.14	0.388	0.419	0.369	0.430	0.368	0.360	0.061
Real Madrid	64.22	0.494	0.360	0.359	0.374	0.454	0.970	0.135
Rosenborg BK	75.41	0.361	0.391	0.261	0.353	0.246	*	0.145
RSC Anderlecht	59.74	0.511	0.432	0.514	0.434	0.403	1.000	0.112
Sparta Prague	57.89	0.589	0.425	0.433	0.421	0.452	*	0.168
Valencia C.F.	88.46	0.115	0.246	0.227	0.250	0.164	*	0.135
Werder Bremen	45.52	0.639	0.543	0.552	0.543	0.558	0.540	0.097

\*: Indeterminate value for the percentage of improvement.

**Table 4.** Efficiency ratios, improvement percentages, and variation ranges: Season 2005-6.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
Ajax	74.19	0.478	0.360	0.258	0.370	0.500	*	0.242
Arsenal (F)	89.86	0.101	0.101	0.344	0.213	0.146	0.230	0.243
Bayern Munich	58.97	0.413	0.528	0.410	0.538	0.500	*	0.128
Benfica	92.09	0.082	0.092	0.083	0.083	0.303	*	0.221
Bruges	100.00							
Chelsea	100.00							
F.C. Barcelona (W)	65.88	0.341	0.341	0.357	0.426	0.438	0.630	0.097
FC Artmedia	100.00							
FC Porto	61.98	0.422	0.402	0.507	0.380	0.464	*	0.127
FC Schalke 04	59.85	0.401	0.450	0.402	0.447	0.436	0.975	0.049
FC Thun	90.79	0.180	0.238	0.092	0.210	0.228	*	0.146
Fenerbahçe	71.67	0.284	0.283	0.410	0.300	0.282	0.880	0.128
Inter Milan	69.34	0.304	0.304	0.304	0.332	0.303	0.680	0.029
Juventus FC	71.33	0.285	0.285	0.291	0.299	0.293	*	0.013
Lille	82.14	0.382	0.267	0.179	0.233	0.429	*	0.251
Liverpool	61.02	0.387	0.387	0.387	0.394	0.436	*	0.050
Lyon	67.47	0.329	0.330	0.434	0.382	0.482	0.340	0.153
Manchester Utd	79.91	0.201	0.325	0.269	0.323	0.201	*	0.124
AC Milan	66.70	0.333	0.333	0.333	0.356	0.333	0.520	0.023
Olympiakos	71.69	0.280	0.281	0.463	0.296	0.373	*	0.182
Panathinaikos	93.00	0.067	0.066	0.270	0.068	0.067	0.600	0.204
PSV Eindhoven	100.00							
Rangers FC	100.00							
Rapid Vienna	77.07	0.227	0.227	0.398	0.241	0.227	0.730	0.171
Real Betis	69.23	0.313	0.312	0.313	0.321	0.371	*	0.059
Real Madrid	56.91	0.434	0.434	0.565	0.448	0.496	*	0.132
Rosenborg BK	89.66	0.101	0.167	0.375	0.179	0.099	0.810	0.276
RSC Anderlecht	96.73	0.041	0.026	0.090	0.116	0.026	*	0.090
Sparta Prague	81.65	0.172	0.219	0.173	0.174	0.241	*	0.069
Udinese	86.22	0.130	0.128	0.128	0.130	0.146	*	0.018
Villarreal	100.00							
Werder Bremen	58.06	0.466	0.419	0.477	0.430	0.507	1.000	0.088

\*: Indeterminate value for the percentage of improvement.

**Table 5.** Efficiency ratios, improvement percentages, and variation ranges: Season 2006-7.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
AC Roma	86.51	0.135	0.135	0.135	0.140	0.190	1.000	0.055
AEK Athens	81.66	0.202	0.186	0.187	0.204	0.186	*	0.018
Arsenal	68.27	0.414	0.322	0.322	0.376	0.322	0.510	0.092
Bayern Munich	78.23	0.331	0.216	0.216	0.262	0.360	*	0.144
Benfica	<b>100.00</b>							
Celtic	<b>100.00</b>							
Chelsea	70.91	0.289	0.290	0.291	0.291	0.292	0.910	0.003
CSKA Moscow	<b>100.00</b>							
F.C. Barcelona	<b>100.00</b>							
Dynamo Kiev	84.28	0.214	0.157	0.157	0.160	0.222	*	0.065
FC Copenhagen	97.55	0.146	0.112	0.031	0.073	0.031	*	0.116
FC Porto	77.24	0.231	0.231	0.232	0.246	0.231	1.000	0.015
FC Shakhtar Donetsk	79.29	0.218	0.207	0.207	0.280	0.362	*	0.155
Galatasaray SK	99.68	0.000	0.242	0.001	0.193	0.048	*	0.242
G. du Bordeaux	89.53	0.104	0.105	0.104	0.149	0.104	*	0.045
Hamburg SV	71.47	0.298	0.285	0.285	0.315	0.368	*	0.083
Inter Milan	<b>100.00</b>							
Levski Sofia	<b>100.00</b>							
Lille	72.06	0.280	0.279	0.278	0.306	0.403	*	0.125
Liverpool (F)	75.01	0.249	0.252	0.250	0.249	0.249	*	0.004
Lyon	85.80	0.224	0.144	0.205	0.145	0.286	*	0.142
Manchester Utd	80.23	0.200	0.200	0.200	0.250	0.327	1.000	0.127
AC Milan (W)	66.27	0.337	0.337	0.338	0.369	0.338	1.000	0.032
Olympiakos	95.27	0.130	0.049	0.089	0.056	0.254	*	0.205
PSV Eindhoven	<b>100.00</b>							
Real Madrid	82.93	0.355	0.167	0.189	0.167	0.310	1.000	0.188
RSC Anderlecht	81.22	0.253	0.180	0.178	0.213	0.180	*	0.075
Spartak Moscow	78.41	0.218	0.220	0.217	0.219	0.219	*	0.003
Sporting Lisbon	70.84	0.288	0.311	0.287	0.286	0.287	*	0.024
Steaua Bucharest	<b>100.00</b>							
Valencia C.F.	93.75	0.059	0.059	0.123	0.059	0.109	*	0.065
Werder Bremen	80.50	0.771	0.805	0.804	0.713	0.685	*	0.120

\*: Indeterminate value for the percentage of improvement.

**Table 6.** Efficiency ratios, improvement percentages, and variation ranges: Season 2007-8.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
Arsenal	67.42	0.418	0.324	0.476	0.393	0.439	*	0.152
Benfica	60.00	0.530	0.400	0.448	0.435	0.564	*	0.164
Besiktas	92.86	0.162	0.077	0.467	0.071	0.170	*	0.395
Celtic	89.66	0.159	0.113	0.182	0.106	0.152	*	0.076
Chelsea (F)	55.98	0.523	0.439	0.440	0.475	0.558	1.000	0.119
CSKA Moscow	76.82	0.232	0.232	0.510	0.312	0.419	0.960	0.279
Dynamo Kiev	63.16	0.426	0.368	0.515	0.394	0.436	*	0.147
F.C. Barcelona	62.61	0.516	0.374	0.644	0.502	0.553	1.000	0.271
Fenerbahçe	66.42	0.502	0.334	0.379	0.335	0.526	*	0.191
G. Rangers	<b>100.00</b>							
Inter Milan	68.44	0.313	0.312	0.421	0.368	0.360	0.860	0.109
Lazio	81.20	0.225	0.188	0.273	0.220	0.343	1.000	0.155
Liverpool	60.18	0.398	0.398	0.549	0.436	0.506	1.000	0.151
Manchester Utd (W)	62.23	0.449	0.377	0.610	0.440	0.497	1.000	0.233
AC Milan	69.90	0.356	0.303	0.481	0.378	0.482	1.000	0.179
O. Lyon	90.00	0.366	0.102	0.481	0.206	0.508	1.000	0.406
O. Marseille	73.81	0.262	0.345	0.515	0.354	0.421	*	0.253
Olympiakos	87.80	0.159	0.124	0.392	0.177	0.286	*	0.268
Oporto	59.72	0.404	0.404	0.559	0.410	0.502	0.905	0.155
PSV Eindhoven	88.89	0.205	0.245	0.111	0.273	0.380	*	0.269
Real Madrid	67.29	0.571	0.329	0.547	0.346	0.582	1.000	0.253
Roma	62.28	0.497	0.376	0.547	0.411	0.517	1.000	0.171
Rosenborg	81.25	0.295	0.206	0.385	0.188	0.333	*	0.197
Schalke 04	66.42	0.472	0.334	0.562	0.353	0.529	*	0.228
Sevilla F.C.	57.83	0.552	0.423	0.468	0.450	0.588	1.000	0.165
Shakhtar Donetsk	52.97	0.470	0.470	0.596	0.497	0.549	0.960	0.126
Slavia Prague	<b>100.00</b>							
Sporting Lisbon	61.26	0.404	0.390	0.543	0.387	0.511	1.000	0.155
Steaua Bucharest	94.12	0.244	0.234	0.059	0.245	0.290	*	0.231
Stuttgart	66.67	0.516	0.379	0.333	0.381	0.500	1.000	0.182
Valencia C.F.	94.12	0.225	0.312	0.059	0.346	0.313	*	0.288
Werder Bremen	51.32	0.530	0.488	0.529	0.487	0.564	1.000	0.078

\*: Indeterminate value for the percentage of improvement.

**Table 7.** Efficiency ratios, improvement percentages, and variation ranges: Season 2008-9.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
Aalborg BK	83.45	0.702	0.180	0.273	0.165	0.197	*	0.536
Anorthosis Fama.	<b>100.00</b>							
Arsenal	69.53	0.747	0.441	0.590	0.474	0.305	1.000	0.442
Atletico Madrid	78.92	0.678	0.216	0.255	0.251	0.213	*	0.465
Basle	85.96	0.702	0.366	0.529	0.373	0.140	*	0.561
Bate Borisov	<b>100.00</b>							
Bayern Munich	56.32	0.787	0.439	0.439	0.488	0.444	0.975	0.348
Celtic	83.78	0.580	0.284	0.204	0.279	0.163	1.000	0.418
Cluj Napoca	99.09	0.752	0.009	0.500	0.033	0.329	*	0.743
Chelsea	66.26	0.774	0.337	0.439	0.378	0.410	*	0.437
Dynamo Kiev	<b>100.00</b>							
F.C. Barcelona (W)	58.90	0.747	0.413	0.587	0.505	0.499	0.905	0.334
Fenerbahçe	88.89	0.735	0.210	0.111	0.221	0.338	*	0.624
Fiorentina	54.77	0.824	0.452	0.579	0.453	0.473	*	0.371
G. de Burdeos	89.34	0.752	0.107	0.158	0.159	0.388	*	0.646
Inter Milan	52.47	0.828	0.477	0.527	0.489	0.491	*	0.351
Juventus	70.20	0.722	0.294	0.419	0.336	0.386	1.000	0.428
Liverpool	64.23	0.739	0.356	0.494	0.375	0.358	0.978	0.383
Manchester Utd (F)	53.19	0.823	0.467	0.610	0.503	0.532	*	0.356
O. Lyon	65.42	0.758	0.347	0.474	0.378	0.446	*	0.410
O. Marseille	57.99	0.735	0.436	0.544	0.455	0.420	*	0.315
Oporto	67.79	0.782	0.321	0.505	0.367	0.465	*	0.461
Panathinaikos	67.23	0.692	0.370	0.467	0.385	0.324	1.000	0.368
PSV Eindhoven	65.05	0.757	0.367	0.348	0.366	0.353	*	0.409
Real Madrid	54.84	0.840	0.453	0.627	0.502	0.585	*	0.387
Roma	82.70	0.724	0.199	0.266	0.229	0.175	*	0.548
Shakhtar Donetsk	73.05	0.646	0.205	0.275	0.207	0.186	*	0.460
Sporting Lisbon	75.25	0.675	0.251	0.251	0.273	0.252	*	0.424
Steaua Bucharest	90.83	0.694	0.092	0.111	0.128	0.125	*	0.602
Villarreal	77.78	0.566	0.225	0.368	0.225	0.336	0.280	0.341
Werder Bremen	46.16	0.828	0.538	0.538	0.539	0.575	0.910	0.290
St. Petersburg	63.64	0.788	0.498	0.660	0.534	0.364	*	0.425

\*: Indeterminate value for the percentage of improvement.

**Table 8.** Efficiency ratios, improvement percentages, and variation ranges: Season 2009-10.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
APOEL Nicosia	<b>100.00</b>							
Arsenal	59.40	0.503	0.405	0.405	0.439	0.405	0.455	0.098
Atletico Madrid	66.13	0.341	0.334	0.422	0.351	0.334	*	0.088
AZ Alkmaar	83.36	0.297	0.169	0.168	0.189	0.171	*	0.129
Bayern Munich (F)	58.33	0.476	0.418	0.503	0.428	0.418	0.415	0.085
Besiktas	77.64	0.300	0.223	0.309	0.229	0.224	*	0.085
Chelsea	63.95	0.377	0.367	0.367	0.371	0.366	*	0.011
CSKA Moscow	<b>100.00</b>							
Debreceni	<b>100.00</b>							
Dynamo Kiev	95.42	0.044	0.046	0.124	0.154	0.200	*	0.156
F.C. Barcelona	57.19	0.445	0.430	0.430	0.467	0.430	*	0.037
Fiorentina	78.32	0.442	0.225	0.220	0.218	0.319	0.220	0.223
G. Rangers	84.74	0.186	0.188	0.152	0.152	0.154	0.590	0.036
G. de Burdeos	70.07	0.423	0.313	0.302	0.302	0.355	0.445	0.121
Inter Milan (W)	94.91	0.091	0.051	0.051	0.070	0.100	*	0.049
Juventus	75.61	0.344	0.201	0.276	0.195	0.315	*	0.149
Liverpool	61.04	0.457	0.390	0.558	0.390	0.390	*	0.169
Maccabi Haifa	92.74	0.224	0.083	0.074	0.072	0.072	*	0.152
Manchester Utd	68.97	0.420	0.316	0.316	0.361	0.316	*	0.105
AC Milan	64.05	0.362	0.368	0.361	0.361	0.392	0.775	0.031
O. Lyon	73.64	0.261	0.265	0.260	0.260	0.305	0.260	0.045
O. Marseille	69.05	0.374	0.377	0.309	0.392	0.311	1.000	0.083
Olympiakos	68.09	0.427	0.339	0.320	0.321	0.320	*	0.107
Oporto	65.31	0.531	0.347	0.347	0.390	0.521	0.350	0.184
Real Madrid	57.54	0.450	0.426	0.614	0.479	0.501	0.420	0.188
Rubin Kazan	99.24	0.055	0.008	0.178	0.084	0.005	*	0.173
Sevilla F.C.	64.67	0.355	0.352	0.351	0.356	0.353	0.350	0.004
Standard Liege	88.59	0.199	0.114	0.142	0.115	0.214	0.110	0.099
Stuttgart	68.82	0.309	0.307	0.307	0.324	0.307	*	0.018
Unirea Urziceni	<b>100.00</b>							
Wolfsburg	54.03	0.525	0.452	0.453	0.454	0.451	0.460	0.074
Zurich	<b>100.00</b>							

\*: Indeterminate value for the percentage of improvement.

**Table 9.** Efficiency ratios, improvement percentages, and variation ranges: Season 2010-11.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
Ajax	79.53	0.333	0.207	0.606	0.205	0.329	*	0.401
Arsenal	81.47	0.199	0.187	0.327	0.260	0.187	1.000	0.140
Auxerre	69.24	0.313	0.338	0.355	0.308	0.308	*	0.047
Basle	57.89	0.421	0.497	0.600	0.481	0.471	*	0.179
Bayern Munich	54.46	0.509	0.457	0.649	0.457	0.518	0.610	0.192
Benfica	57.73	0.423	0.434	0.729	0.423	0.456	*	0.307
Bursaspor	75.24	0.308	0.248	0.473	0.285	0.247	*	0.226
Cluj Napoca	87.83	0.182	0.170	0.435	0.122	0.209	*	0.313
Chelsea	62.15	0.474	0.381	0.395	0.406	0.534	0.380	0.153
Copenhagen	89.80	0.104	0.184	0.379	0.192	0.235	1.000	0.275
F.C. Barcelona (W)	90.45	0.406	0.094	0.332	0.239	0.458	0.277	0.363
FK Partizan	86.32	0.182	0.200	0.381	0.137	0.264	1.000	0.244
G. Rangers	<b>100.00</b>							
Hapoel Tel Aviv	<b>100.00</b>							
Inter Milan	94.02	0.377	0.058	0.411	0.165	0.455	0.060	0.398
Manchester Utd (F)	69.88	0.349	0.303	0.608	0.348	0.303	1.000	0.305
AC Milan	86.32	0.306	0.139	0.578	0.139	0.199	*	0.439
MSK Zilina	68.75	0.313	0.338	0.533	0.317	0.361	*	0.221
O. Lyon	54.64	0.455	0.459	0.692	0.455	0.497	1.000	0.237
O. Marseille	50.63	0.495	0.495	0.638	0.500	0.509	*	0.143
Panathinaikos	<b>100.00</b>							
Real Madrid	80.73	0.521	0.193	0.534	0.330	0.513	0.190	0.342
Roma	74.57	0.409	0.256	0.558	0.256	0.414	0.833	0.302
Rubin Kazan	88.19	0.118	0.125	0.479	0.118	0.193	1.000	0.361
Schalke 04	69.85	0.305	0.313	0.624	0.302	0.301	*	0.323
Shakhtar Donetsk	81.06	0.188	0.188	0.609	0.214	0.201	1.000	0.422
Spartak Moscow	66.35	0.337	0.338	0.572	0.336	0.337	*	0.235
Sporting Braga	<b>100.00</b>							0.000
Tottenham Hotspur	82.09	0.177	0.280	0.443	0.315	0.289	1.000	0.266
Twente	58.87	0.411	0.418	0.582	0.411	0.491	1.000	0.170
Valencia C.F.	51.86	0.482	0.479	0.685	0.479	0.479	1.000	0.206
Werder Bremen	51.46	0.508	0.486	0.662	0.497	0.485	*	0.177

\*: Indeterminate value for the percentage of improvement.



**Table 10.** Efficiency ratios, improvement percentages, and variation ranges: Season 2011-12.

	Efficiency %	Improvement percentage						Variation range
		Assists	Centers	Corners	Opponent's area	Shots	Penalties	
Ajax	55.35	0.562	0.447	0.578	0.447	0.449	*	0.131
APOEL Nicosia	<b>100.00</b>							
Arsenal	84.71	0.219	0.404	0.370	0.383	0.154	0.160	0.250
Basle	75.00	0.288	0.457	0.250	0.380	0.250	0.250	0.207
Bate Borisov	<b>100.00</b>							
Bayer Leverkusen	81.82	0.291	0.357	0.412	0.331	0.184	*	0.228
Bayern Munich (F)	47.86	0.670	0.521	0.554	0.548	0.614	0.520	0.149
Benfica	58.06	0.551	0.481	0.453	0.469	0.418	*	0.133
Borussia Dortmund	60.32	0.397	0.529	0.397	0.499	0.419	0.400	0.133
Chelsea (W)	62.49	0.138	0.127	0.036	0.160	0.124	0.373	0.124
CSKA Moscow	82.76	0.291	0.259	0.278	0.235	0.174	*	0.116
Dynamo Zagreb	<b>100.00</b>							
F.C. Barcelona	59.04	0.665	0.410	0.595	0.544	0.670	0.600	0.261
Inter Milan	65.79	0.401	0.362	0.456	0.339	0.349	0.340	0.118
Lille	56.25	0.543	0.635	0.635	0.604	0.438	*	0.198
Manchester City	63.49	0.389	0.416	0.367	0.429	0.366	0.370	0.064
Manchester United	60.11	0.441	0.618	0.401	0.622	0.400	0.450	0.222
AC Milan	81.73	0.184	0.349	0.186	0.391	0.213	0.185	0.207
Napoli	67.92	0.413	0.479	0.482	0.443	0.319	0.320	0.163
O. Lyon	56.30	0.523	0.535	0.438	0.508	0.492	*	0.096
O. Marseille	72.58	0.341	0.605	0.306	0.566	0.277	0.280	0.329
Olympiakos	73.08	0.317	0.429	0.269	0.384	0.306	*	0.160
Oporto	40.85	0.590	0.628	0.592	0.591	0.652	0.590	0.062
Otelul Galati **	<b>100.00</b>	0.273	0.482	0.000	0.432	0.182	*	0.482
Racing Genk	91.53	0.238	0.298	0.406	0.254	0.085	*	0.322
Real Madrid	55.80	0.640	0.441	0.479	0.486	0.589	0.440	0.199
Shakhtar Donetsk	51.92	0.560	0.518	0.565	0.504	0.479	0.480	0.086
Trabzonspor	74.83	0.262	0.407	0.252	0.307	0.251	0.250	0.156
Valencia C.F.	54.11	0.529	0.524	0.462	0.516	0.462	0.470	0.067
Viktoria Plzen	79.41	0.360	0.270	0.424	0.307	0.206	*	0.218
Villarreal	94.81	0.111	0.052	0.136	0.083	0.069	*	0.084
Zenith St. Petersburg	62.45	0.453	0.427	0.375	0.409	0.375	0.380	0.077

\*: Indeterminate value for the percentage of improvement.

\*\*: This is a team whose efficiency ratio is 1, but with positive slack.

The results show that in seasons 2007-8, 2008-9, 2010-11, and 2011-12, a considerable number of teams had improvement percentages spread over a wide range. In 2008-9, all the teams classified as inefficient show a range of variation greater than 0.15, and the improvement percentage for *assists* is greater than the rest of the productive factors. Consequently, this is the one that should be reduced to the greatest extent.

In 2010-11, 24 of the 28 teams classified as inefficient exceeded the variation range limit, and for 22 of these, *corner kicks* has the highest improvement percentage; it was *shots* in the remaining two cases. In 2007-8 and 2011-12, the number of teams whose improvement percentage varied considerably is more similar to those found in the opposite case. In the former, 19 inefficient teams had a variation range greater than 0.15

(compared to 11 with a lower value), of which nine had *corners* as the input which should be reduced by a proportion greater than the other resources. Eight had *shots*, one had *assists*, and another had *entries in the opponent's area*. For 2011-12, 13 teams had a wide variation (compared to 16 that did not), and all productive resources had to be reduced in some cases (for five teams it was *centers*; for four, *corners*; for two, *entries in the opponent's area*; for one, *shots*; and for another one, *assists*).

In the rest of the seasons, the number of teams that should change their way of playing to become efficient is lower, and varies from three teams in 2006-7 to nine in 2004-5. In these seasons, four teams appear at least twice in this situation. Galatasaray had a range of variation in its improvement percentage greater than 0.15 in 2003-4 and 2006-7, and in both cases *centers* and *entries in the opponent's area* were the resources to reduce in the greatest proportion.

Oporto shows a similar trend for the 2004-5 and 2009-10 seasons, with *assists* and *shots* showing an improvement percentage higher than the rest. On the other hand, Olympiakos had to reduce *corners* and *shots* by a greater percentage for 2003-4 but only *corners* in 2005-6 and only *shots* in 2006-7. Real Madrid did not show a consistent trend either: *assists* needed to be reduced the most for 2006-7, while in 2009-10 it was *corners*. For the other teams showing a high range of variation in the improvement percentage in the 2003-4, 2004-5, 2005-6, 2006-7, and 2009-10 seasons, no particular input required a greater reduction in use compared to the rest. *Corners* and *assists* were the most common, followed by *shots*, while *entries in the opponent's area* and *centers* were the least frequent.

As for *penalties*, the reference units for calculating the inefficiency ratio for most of the teams in this study present a null value.<sup>12</sup> This situation is reflected both in indeterminate improvement percentages (the reference and real values are zero) and for values equal to unity (because the number of penalties in the reference unit is zero, the numerator and denominator of the improvement percentage coincide).

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<sup>12</sup> Although reference unit is not always a real DMU but is the linear combination of data coming from efficient units, it is realistic that a football team presents a number of penalties equal to zero during the whole competition.

## 5. CONCLUSIONS

The aim of this study is to improve the efficiency of European football teams participating in the UEFA Champions League between 2003 and 2012. The data refer to the season as a whole, and each season was analyzed separately. The study focuses on the activity on the field—the second of two stages of the production for sports teams.

We use DEA to calculate efficiency. It estimates the isoquant as a production frontier by solving linear programming problems. The variables representing inputs and results are based on proposals for isoquants found in economic theory. Accordingly, offensive plays are inputs, and games played in competition are the results.

Our study identifies the efficiency levels of the football teams in the sample, together with the input and output levels of the team situated on the isoquant that serves as a reference for calculating efficiency ratios. By comparing the input amounts the reference team uses to the amounts actually consumed by the team under study, we calculate an improvement percentage that indicates, for each input, how much the team must reduce its use of a particular resource in order to become efficient. We interpret similar improvement percentages for all of a team's inputs as an indication that the team has a playing style similar to that of the reference unit, but that use of all resources should be reduced in similar proportions to improve efficiency. On the other hand, when one or two production factors have improvement percentages substantially higher than the rest, this indicates that the team should change its game to reduce the use of these factors in a greater proportion than the others. Furthermore, if a production factor consistently has significantly higher improvement percentages than the rest, we can make very general recommendations for all inefficient teams. The results of this study show that in 2008-9, inefficient teams wasted assists in higher proportions than other production factors. However, this homogeneity is not seen in the other seasons analyzed.

Taken individually, some teams with large ranges of variation in more than one season, which seldom occurs, tend to show a stable situation over time (e.g., Galatasaray and Oporto) while others do not (e.g., Olympiakos and Real Madrid). In short, no single input needs more reduction than the others. Therefore, inefficient teams should use fewer offensive plays of all types; though we cannot determine a change in the style of game.

This recommendation does not mean teams should only reduce plays; in the DEA approach, an inefficient organization improves its efficiency by reducing the amount of

inputs but maintaining the same level of output. Therefore, this study shows that many teams in the UEFA Champions League could have gotten the same results with fewer offensive plays<sup>13</sup>, and the reduction would have been the result changing the work done in training sessions.

Finally, recommending a reduction in attacking moves could be interpreted as focusing exclusively on playing tactics; however, there is an economic angle. In the second stage of the production process for football teams, cost savings from reducing the number of attacking moves during games are not readily discernible. However, attacking moves come from transforming the production factors in the first stage (during training sessions and pre-game preparation), and these inputs do have an easily calculable cost. A reduction in the need for attacking plays reduces the required external inputs, thus reducing costs.

This study includes two variables over which football teams do not have complete control as resources, and which merit some additional comments. These are corners and penalties. Although we find that none of the inputs considered need reduction at a greater percentage than the rest, corner kicks do appear frequently in this situation in our sample. So, this play scarcely contributes to obtaining output. Therefore, we cannot conclude that a football team's results depend directly on the performance of the opponent. The conclusion is the same for penalties because, in many cases, the reference units for the inefficient teams show a null value for this variable (i.e., it is not necessary to make use of this resource to be situated on the isoquant).

In short, teams classified as inefficient rarely use corner kicks and penalties (in the case of penalties, even the recommendation for efficiency would include null values for this resource), indicating that the offensive plays that are a result of the opponent's moves are not relevant to the efficiency of football teams.

In this paper, we choose DEA to calculate efficiency because it does not need a specification of production functions. This can be an advantage in the case of football teams, which use a labor-intensive technology to get results. Also, the continuous adaptation of play in the field prevents building a generalized production function for these organizations. These same circumstances make it difficult to recommend exact amounts of inputs inefficient teams should use to become efficient.

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<sup>13</sup> This was true also for the champions and losing finalists in the seasons studied, as we found no case of an efficiency ratio equal to unity for them.

This paper overcomes this limitation by calculating improvement percentages and analyzing styles of play instead counting the exact number of moves needed. Nevertheless, the lack of a mathematical function representing the productive process for football teams is a real limitation that also affects DEA in its original formulation. Productive process is, in consequence, a “black box” that is hard to analyze. However, other studies of both stages of production for football teams (training and matches) could show new ways to improve efficiency. Network DEA, which calculates efficiency using as inputs the output variables in the previous stage, could be one possible tool to detect possible areas of improvement for inefficient teams.

Our paper is also framed by a global context in which teams aim to be league champions. Other approaches could analyze how previous matches affect the style of play or could also divide the results from different stages of competition and to study the influence one stage has in the next. Again, network DEA can be useful for this analysis, though it only can be done for teams reaching the last stage, thereby reducing the sample.

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